Gravitational Lensing

Stefan Brems

Astronomy Summerschool 2013
Ruprecht-Karls Universität Heidelberg

23. August 2013

Content

Development

- Soldner calculated deflection of the light by assuming it as massive particles 1804
- 1911 Einstein employed the equivalence principle and re-derived Soldner's formula
- in 1915 Einstein applied the full field equations of General Relativity and discovered twice the deflection angle caused by space-time distortion
- this predicted a deflection of 1"7 of light grazing tangentially the sun's surface →verified in 1919 during the eclipse of the sun

Deflection of light rays

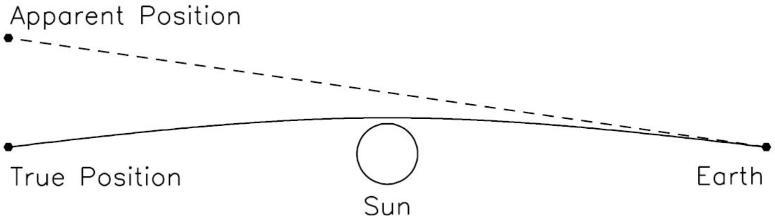


Figure : Angular deflection of a ray passing close to the limb of the sun

- 1920 Eddington noted the possibility of multiple lightpaths connecting a source and an observer
  →multiple images of a single light source (although the chance to observe was assumed very little)
- 1937 Zwicky pointed out that galaxies could do that and also magnify distant galaxies

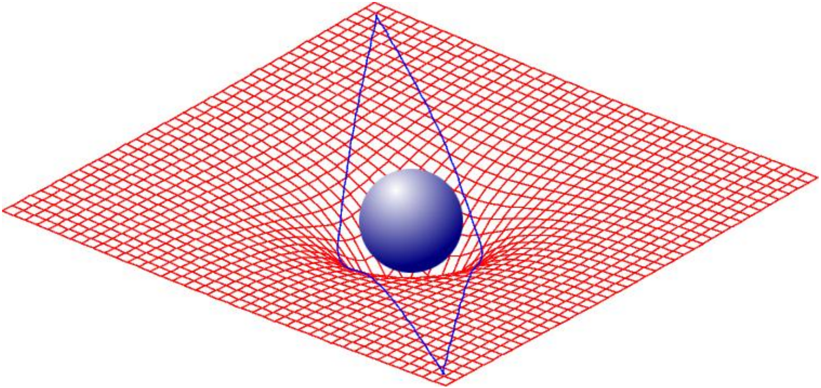


Figure : Illustrated, perturbed spacetime

Deflection by Pointmasses

We're assuming a Friedmann-Lemaître-Robertson-Walker-metric (homogeneous and isotropic) with only local perturbations
→locally Minkowskian spacetime

- the refraction index n can now be approximated by the Newtonian Potential Φ as

$$n \approx 1 - \frac{2}{c^2}\Phi \overset{\Phi \leq 0}{=} 1 + \frac{2}{c^2}|\Phi|$$

- the speed of light changes to

$$v = \frac{c}{n} \approx c - \frac{2}{c}|\Phi|$$

We can compare this to the deflection by a glass prism:

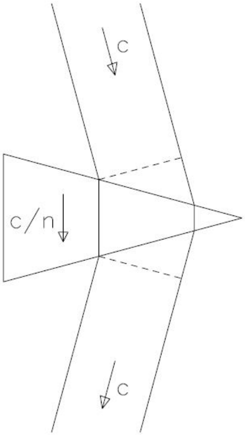


Figure : Light being deflected by a prism

We can now calculate the time delay of the light signal, the so called *Shapiro Delay* (1964):

$$\Delta t = \int_{source}^{observer} \frac{2}{c^3} |\Phi| dt$$

As also the *deflection angle* $\hat{\alpha}$:

$$\vec{\hat{\alpha}} = -\int \vec{\nabla}_{\perp} n dl = \frac{2}{c^2} \int \vec{\nabla}_{\perp} \Phi dl$$

But as $\hat{\alpha}$ is always very small we can just integrate along an unperturbed light ray

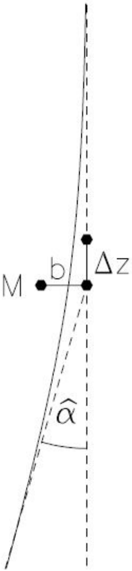
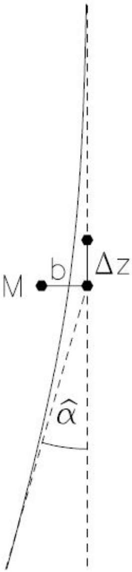


Figure : Light deflection by a point mass. The unperturbed light ray passes the mass at impact parameter b and is deflected by the angle $\hat{\alpha}$



- we get the Newtonian potential of the point mass M as

$$\Phi(b,z) = -\frac{GM}{(b^2+(\Delta z)^2)^{1/2}}$$

$$\Rightarrow \quad \vec{\nabla}_\perp \Phi(b,z) = \frac{GM\vec{b}}{(b^2+(\Delta z)^2)^{3/2}}$$

- using this to calculate $\hat{\alpha}$ we get

$$\boxed{\hat{\alpha} = \frac{2}{c^2}\int \vec{\nabla}_\perp \Phi \, dz = \frac{4GM}{c^2 b}} \qquad (1)$$

which is simply twice the inverse of the impact parameter in units of the Schwarzschildradius $R_S = \frac{2GM}{c^2}$

Thin screen Approximation

As the lens is comparably thin to the total extend of the light path we will consider it as a *lens plane* which is characterized by its surface mass density Σ:

$$\Sigma(\vec{\xi}) = \int \rho(\vec{\xi}, z)\, dz$$

Our formula (1) will change to

$$\vec{\hat{\alpha}}(\vec{\xi}) = \frac{4G}{c^2} \int \frac{(\vec{\xi} - \vec{\xi'})\Sigma(\vec{\xi'})}{|\vec{\xi} - \vec{\xi'}|^2} d^2\xi',$$

where the deflection angle $\vec{\hat{\alpha}}$ at position $\vec{\xi}$ is due to the sum of all mass elements in the plane.

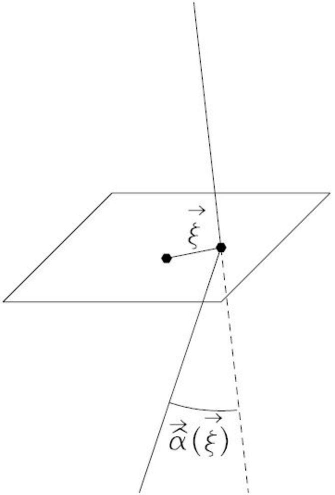


Figure : Illustration of the thin screen approximation

If we assume the lens being symmetric, we can shift the coordinate origin to the center of symmetry, so we will get the easier formula

$$\vec{\hat{\alpha}}(\vec{\xi}) = \frac{4GM(\vec{\xi})}{c^2 \xi}$$

with

$$M(\xi) = 2\pi \int_0^{\xi} \Sigma(\xi')\xi' \, d\xi'$$

Lensing Geometry

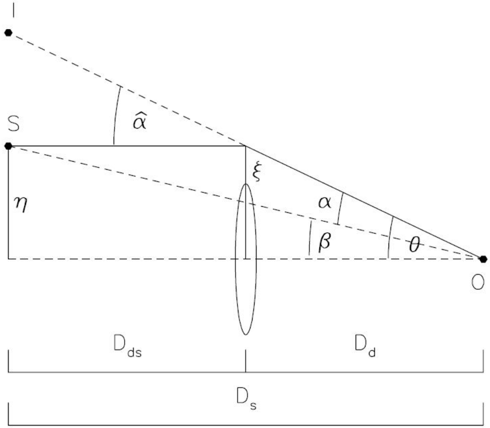
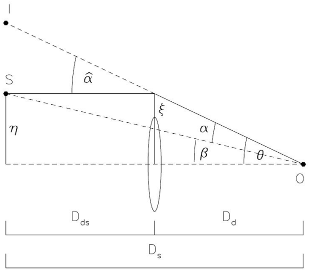


Figure : Illustration of a gravitational lens system from Source S to Observer O



We define the *reduced deflection angle*

$$\vec{\alpha} = \frac{D_{ds}}{D_s}\vec{\hat{\alpha}}$$

and the *lens equation*:

$$\vec{\beta} = \vec{\theta} - \vec{\alpha}(\vec{\theta}). \qquad (2)$$

We now assume a constant surface mass density. (1) gives:

$$\alpha(\theta) = \frac{D_{ds}}{D_s}\frac{4G}{c^2\xi}\underbrace{(\Sigma\pi\xi^2)}_{M(\xi)} = \frac{4\pi G\Sigma}{c^2}\frac{D_d D_{ds}}{D_s}\theta$$

where we set $\xi = D_d\theta$.

We can now also define the *critical surface mass density* Σ_{cr}, where $\alpha(\theta) = \theta \quad \Leftrightarrow \beta = 0$

$$\Sigma_{cr} = \frac{c^2}{4\pi G} \frac{D_s}{D_d D_{ds}} = 0.35 \frac{g}{cm^2} \left(\frac{1\,Gpc \cdot D_s}{D_d D_{ds}} \right)$$

The lens so focuses perfectly with a defined focal length.
A lens with $\Sigma > \Sigma_{cr}$ is called *supercritical*.

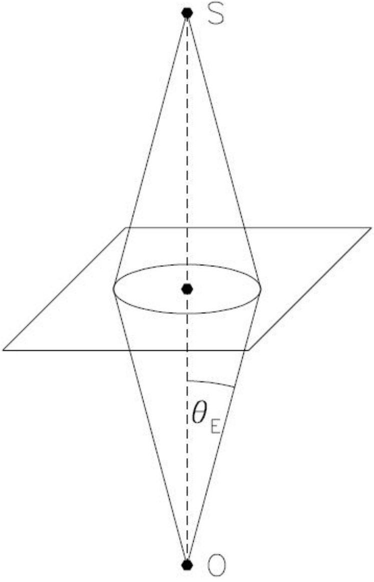
Considering now a circularly symmetric lens with arbitrary mass profile. The lens equation leads to

$$\beta(\theta) = \theta - \frac{D_{ds}}{D_d D_s} \frac{4GM(\theta)}{c^2 \theta}$$

If we now have a source lying exactly on the optic axis, it is imaged as a ring if the lens is supercritical. By setting $\beta = 0$ we obtain the *Einstein radius* of the ring:

$$\theta_E = \left(\frac{4GM(\theta_E)}{c^2} \frac{D_{ds}}{D_d D_s} \right)^{\frac{1}{2}} \tag{3}$$

Einstein ring



Considering a source not exactly on the symmetry line.
Rewriting the lens equation (2) for a point mass as

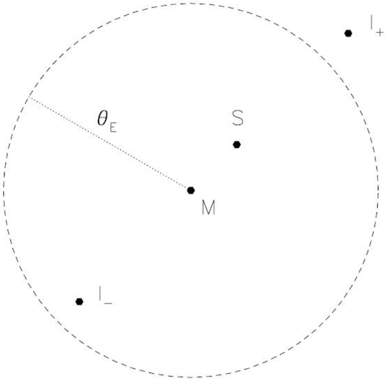
$$\beta = \theta - \frac{\theta_E^2}{\theta}. \tag{4}$$

with the two solutions

$$\theta_\pm = \frac{1}{2}\left(\beta \pm \sqrt{\beta^2 + 4\theta_E^2}\right)$$

→every source is imaged twice →one image on each side of the source →one inside, one outside the Einstein ring

Einstein ring



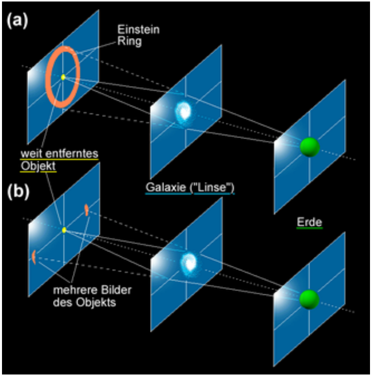


Figure : Visualization of multiple images (left) and Einstein rings (right). Source: Wikipedia

Magnification

- Gravitational light deflection preserves surface brightness
- but Gravitational Lensing can change the flux by changing the apparent solid angle of a source of a source as

$$magnification = \frac{imagesize}{sourcearea}$$

or in formula

$$\mu = \frac{\theta}{\beta}\frac{d\theta}{d\beta}$$

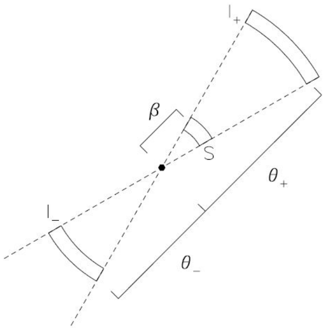


Figure : Magnified image of a source lensed by a point mass.

We can calculate the magnification by using (4) and its values for $\mu_\pm$ (only for pointmasses):

$$\mu_\pm = \left[1 - \left(\frac{\theta_E}{\theta_\pm}\right)^4\right]^{-1}$$

$$= \frac{u^2+2}{2u\sqrt{u^2+4}} \pm \frac{1}{2}$$

where u is the angular seperation of source and point mass in units of the Einstein angle: $u = \beta\theta^{-1}$.

Usually these two circles images are not resolvable, because the typical order of magnitude of the Einstein radius is:

$$\theta_E = (0.9") \left(\frac{M}{10^{11} M_\odot} \right)^{\frac{1}{2}} \left(\frac{D}{Gpc} \right)^{-\frac{1}{2}}$$

So we define the *total magnification*:

$$\mu := |\mu_+| + |\mu_-| = \frac{u^2 + 2}{u\sqrt{u^2 + 4}} \overset{u=1}{=} 1.17 + 0.17 = 1.34$$

This fluctuation can be observed if lens and source move relatively to each other.

Thank you for your attention