dark matter haloes and galaxy formation

cosmology lecture (chapter 12)

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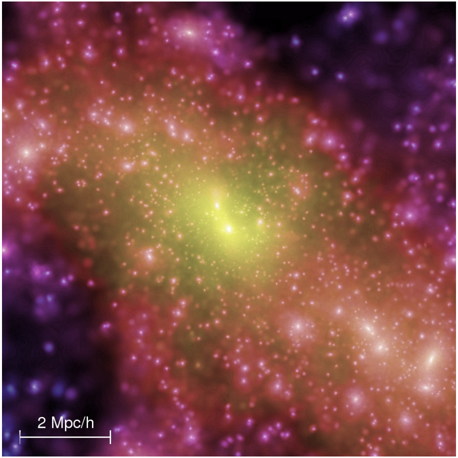
August 1, 2013

outline

repetition

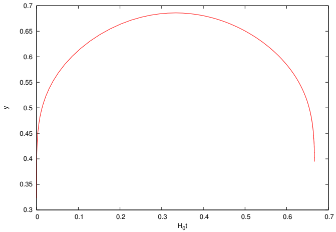
- Friedmann-Lemaître cosmologies with matter and dark energy for accelerated expansion
- thermal history of the universe explains element synthesis and the microwave background
- inflation needed for solving the flatness and horizon-problems, and provides Gaussian initial fluctuations for structure growth
- growth is linear and homogeneous initially, and conserves the Gaussianity of the fluctuation field
- later, growth becomes inhomogeneous and nonlinear, destroys Gaussianity by mode coupling
- galaxy rotation can be explained by tidal torquing, linear flows are necessarily laminar
- fluid dynamics with dark matter is special:
    - gravity is infinitely reached
    - collisionlessness → no pressure, no viscosity

nonlinearly evolved density field



source: V.Springel, Millenium simulation

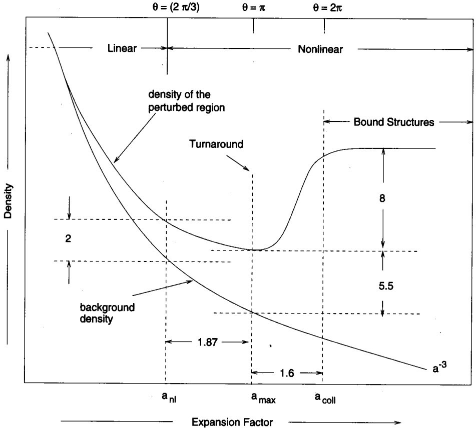
spherical collapse



source: F.Pace, collapse in SCDM

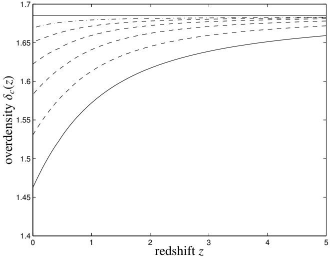
- formation of a bound dark matter object: gravitational collapse
- three phase process:
  1. perturbation expands with Hubble expansion, but at a lower rate
  2. perturbation decouples from Hubble expansion → turn around
  3. perturbation collapses under its own gravity

density evolution in a collapsing halo



source: Padmanabhan, theoretical astrophysics

collapse overdensity in different cosmologies



overdensity needed for a perturbation to collapse at redshift z

- SCDM: collapse overdensity of $\delta_c = 1.686$, very similar in ΛCDM
- dark energy cosmologies require **smaller** collapse overdensities
- sensitivity towards dark energy parameters

relaxation

- in the dynamical evolution, systems tend towards a final state which is not very sensitive on the initial conditions → **relaxation**
- usually, this is accompanied by generation of entropy, which defines an arrow of time
- in cosmology, galaxies with very similar properties form from a Gaussian fluctuation in the matter distribution
- but: dark matter is a collisionless fluid!
  - no viscosity in Euler-eqn. which can dissipate velocities
  - transformation from kinetic energy to heat is not possible
  - no Kelvin-Helmholtz instability and Kolmogorov cascading
  - Euler-equation is time-reversible and no entropy is generated
  - relaxation does not take place

question

show that the Euler-eqn. and the vorticity eqn. are time-reversible

relaxation: 1. two-body relaxation

two-body relaxation

relaxation with **Keplerian** (time-reversible) orbits in a succession of two-body encounters

- consider a system with N stars of size R, density of stars is $n \sim N/R^3$, total mass $M = Nm$
- shoot a single star into the cloud an track its transverse velocity
- in a single encounter the velocity changes

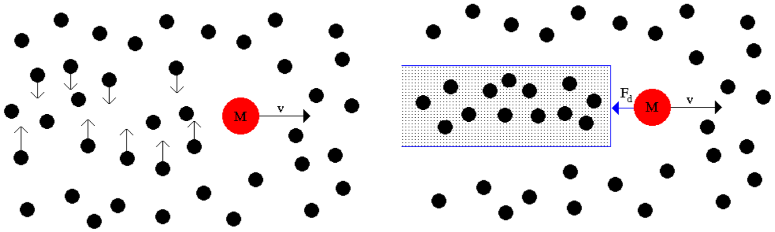
$$\delta\upsilon_\perp(\text{single}) \sim \frac{Gm}{b^2}\frac{2b}{\upsilon} \sim \frac{2Gm}{b\upsilon} \tag{1}$$

with impact parameter b, using Born-approx. with $\delta t = 2b/\upsilon$

- multiple encounters: add random kicks, so variance $\delta\upsilon_\perp^2$ grows

$$\frac{\mathrm{d}}{\mathrm{d}t}\delta\upsilon_\perp^2 \sim 2\pi\int b\mathrm{d}b\,\delta\upsilon_\perp(\text{single})\,n\upsilon = \frac{8\pi G^2 m^2 n}{\upsilon}\ln\left(\frac{b_{\max}}{b_{\min}}\right) \tag{2}$$

relaxation: 2. dynamical friction



source: J. Schombert

- system of reference with moving particle
- all other particle zoom past on hyperbolic orbits, orbit/gravitational scattering depends sensitively on the impact parameter
- directed, ordered velocities → random transverse velocities

relaxation: 3. violent relaxation

- proposed by Lynden-Bell for explaining the brightness profile of elliptical galaxie, wipes out structure of spiral galaxies in the merging
- each particle sees a rapidly fluctuating potential generated by all particles

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \frac{m}{2}\frac{\mathrm{d}\upsilon^2}{\mathrm{d}t} + \frac{\partial\Phi}{\partial t} + \vec{\upsilon}\nabla\Phi \tag{3}$$

- dynamic kind of scattering mediated by grav. field

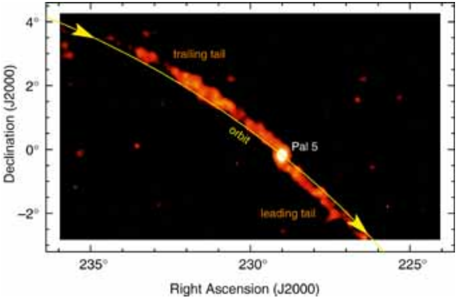
$$\text{with} \quad \frac{\mathrm{d}\upsilon^2}{\mathrm{d}t} = 2\vec{\upsilon}\frac{\mathrm{d}\vec{\upsilon}}{\mathrm{d}t} = -\frac{2}{m}\vec{\upsilon}\nabla\Phi \quad \rightarrow \quad \frac{\mathrm{d}E}{\mathrm{d}t} = \frac{\partial\Phi}{\partial t} \tag{4}$$

- even particles with initially similar trajectories get separated

violent relaxation

important relaxation mechanism, due to long-reaching gravity

relaxation: 4. phase space mixing



globular cluster Palomar-5, source: J. Staude

- time evolution of a globular cluster orbiting the Milky Way:
  - stars closer to Galactic centre move faster
  - stars further away move slower
- with time, the streams get more elongated and eventually form a tightly wound spiral

relaxation: 4. phase space mixing

- naive interpretation:
  system produces structure on smaller and smaller scales (spiral winds up), eventually crosses thermodynamic scale λ
- but: the system is time-reversible and does conserve full phase space information
- relaxation does not take place, the system remembers its initial conditions
- thermodynamic scale is not well defined, gravity is a power law!
- solution: no matter how small the thermodynamic scale is chosen, the system will always wipe out structures above this scale with time $\rightarrow$ **coarse-graining**

generation of entropy

phase space density f measured above this scale decreases, and entropy $S \propto -\int \mathrm{d}^3p\mathrm{d}^3q\, f \ln f$ increases

final state: virialisation

final state

relaxation mechanisms generate a final state which does not depend on the initial conditions, e.g. a stable galaxy from some random flucutation in the Gaussian density field

- a virialised object does not evolve anymore and is characterised by a symmetric phase space distribution → **equipartition**, and a velocity distribution which depends only on constants of motion
- systems are stabilised against gravity by their particle motion, despite the lack of a microscopic collision mechanism which provides pressure
- virial relation $2\langle T\rangle = -\langle V\rangle$ between mass, size and temperature

$$\langle \upsilon^2\rangle = 3\sigma_\upsilon^2 = \frac{GM}{R} \rightarrow M \simeq \frac{3R\sigma_\upsilon^2}{G} = 10^{15} M_\odot/h \left(\frac{R}{1.5\mathrm{Mpc}/h}\right)\left(\frac{\sigma_\upsilon}{1000\mathrm{km}/s}\right)^2 \quad (5)$$

stability: density profiles of dark matter objects

- does a final state exist? needs to maximise entropy...
- use phase space density f for describing the steady-state distribution of particles in a dark matter halo
- solution need to be a solution of the collisionless steady-state ($\partial f/\partial t = 0$) Boltzmann-eqn.

$$\frac{\mathrm{d}f}{\mathrm{d}t} = \frac{\partial f}{\partial t} + \vec{\upsilon}\nabla_x f - \nabla\Phi\nabla_\upsilon f = 0 \tag{6}$$

- and they need to be self consistent: the mass distribution generates its own potential

$$\Delta\Phi = 4\pi G\rho \text{ with } \rho = m\int \mathrm{d}^3\upsilon\, f(\vec{x},\vec{\upsilon}) \tag{7}$$

- originally for galactic dynamics, applies for dark matter as well (collisionlessness)

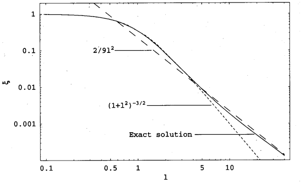
self-consistent solutions of dark matter objects

- Ansatz for phase space density f: should depend on the integrals of motion C, because then f satisfies the steady-state Boltzmann-equation: $\mathrm{d}f/\mathrm{d}t = \partial f/\partial C \times \partial C/\partial t$
- shift potential Φ: $\psi = -\Phi + \Phi_0$, with constant Φ_0 (make ψ vanish at boundary)
- simple approach: phase space density $f(\vec{x}, \vec{v})$ depends only on $\epsilon = \psi - v^2/2$, assumption of spherical symmetry
- matter density ρ for a model follows from

$$\rho(\vec{x}) = \int_0^{\psi} \mathrm{d}\epsilon\, 4\pi f(\epsilon)\, \sqrt{2(\psi - \epsilon)} \tag{8}$$

- substitute ρ in Poisson equation: $\Delta\psi = -4\pi G\rho$, solve for ψ as a function of ϵ, boundary conditions on $\psi(0) = \psi_0$ and $\psi'(0) = 0$

singular isothermal sphere



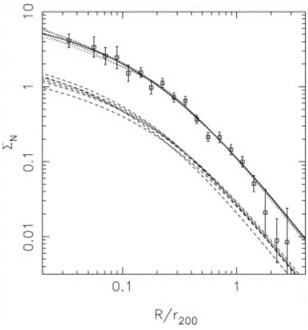
credit: Padmanabhan, theoretical astrophysics

- distribution function, motivated by Boltzmann statistics

$$f(\epsilon) = \frac{\rho_0}{(2\pi\sigma^2)^{3/2}} \exp\left(\frac{\epsilon}{\sigma^2}\right) \quad (9)$$

- properties:
  - constant velocity dispersion inside object, $\sigma^2 = 3\langle \upsilon^2 \rangle$
  - temperature assignment $k_B T \propto \sigma^2$
  - numerical solution to Boltzmann-problem exists, finite core density
  - at large radii, $\rho \propto r^{-2} \rightarrow$ flat rotation curve

Navarro-Frenk-White profile



question

construct a possible fitting formula for the NFW-profile!

Navarro-Frenk-White profile

- Navarro, Frenk + White: haloes in n-body simulation show a profile:

$$\rho \propto \frac{1}{x(1+x^2)} \quad \text{with} \quad x \equiv \frac{r}{r_c} \quad \text{and} \quad r_c = c r_{\mathrm{vir}} \tag{10}$$

- universal density profile, applicable to haloes of all masses
- fitting formula breaks down:
  - infinite core density
  - total mass diverges logarithmically
- very long lived transitional state (gravothermal instability)
- scale radius r_s is related to virial radius by concentration parameter c
- c has a weak dependence on mass in dark energy models

question

show that the NFW-profile allows flat rotation curves! what's the size of the galactic disk? what happens if the disk is very large?

number density of collapsed objects

halo formation

haloes form at peaks in the density field → reflect the fluctuations statistics in the high-δ tail of the probability density

- valuable source of information on Ω_m, σ_8, w and h
- prediction of the number density of haloes from the spectrum $P(k)$ → **Press-Schechter formalism**
- relate mass M to a length scale R

$$M = \frac{4\pi}{3}\Omega_m \rho_{\mathrm{crit}} R^3 \tag{11}$$

- how often does the density field try to exceed some threshold δ_c on the mass scale M?

Press-Schechter formalism

- consider variance of the convolved density field

$$\sigma_R^2 = \frac{1}{2\pi^2}\int \mathrm{d}k\, k^2 P(k) W(kR)^2 \tag{12}$$

  with a top-hat filter function of size R

- convolved field $\bar{\delta}$ has a Gaussian statistic with the variance σ_R^2

$$p(\bar{\delta}, a)\mathrm{d}\bar{\delta} = \frac{1}{\sqrt{2\pi\sigma_R^2}} \exp\left(-\frac{\bar{\delta}^2}{2\sigma_R^2(a)}\right) \tag{13}$$

  with $\sigma_R^2(a) = \sigma_R^2 D_+(a)$

- condition for halo formation: $\bar{\delta} > \delta_c$
- fraction of cosmic volume filled with haloes of mass M

$$F(M, a) \int_{\delta_c}^{\infty} \mathrm{d}\bar{\delta}\, p(\bar{\delta}, a) = \frac{1}{2}\mathrm{erfc}\left(\frac{\delta_c}{\sqrt{2}\sigma_R(a)}\right) \tag{14}$$

Press-Schechter formalism

- distribution of haloes with mass M: $\partial F(M)/\partial M \rightarrow$ add relation between M and R

$$\frac{\partial F(M)}{\partial M} = \frac{1}{\sqrt{2\pi}} \frac{\delta_c}{\sigma_R D_+(a)} \frac{\mathrm{d}\ln\sigma_R}{\mathrm{d}M} \exp\left(-\frac{\delta_c}{2\sigma_R^2 D_+^2(a)}\right) \tag{15}$$

after using the derivative

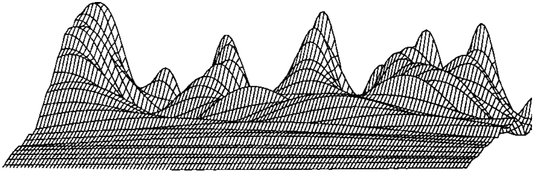
$$\frac{\mathrm{d}}{\mathrm{d}x}\mathrm{erfc}(x) = -\frac{2}{\sqrt{\pi}}\exp(-x^2) \tag{16}$$

- comoving number density: divide occupied cosmic volume fraction by halo volume M/ρ_0

$$n(M,a)\mathrm{d}M = \frac{\rho_0}{\sqrt{2\pi}} \frac{\delta_c}{\sigma_R D_+(a)} \frac{\mathrm{d}\ln\sigma_R}{\mathrm{d}\ln M} \exp\left(-\frac{\delta_c^2}{2\sigma_R^2 D_+^2(a)}\right) \frac{\mathrm{d}M}{M^2} \tag{17}$$

- normalisation is not right by a factor of 2, but there is an elaborate argument for fixing it

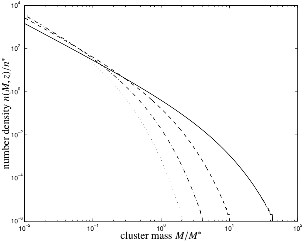
halo formation as a random walk



source: Bond et al. (1991)

- if the density is smoothed with $R = \infty$, the mean density of any perturbation is $\delta = 0$ and $\rho = \bar{\rho} = \Omega_m \rho_{\mathrm{crit}}$
- reduce filter scale: density field will develop fluctuations
- if a density on scale R exceeds the threshold δ_c, it will collapse and form an object of mass $M = 4\pi\rho_0\delta R^3/3$
- at a single point in space: δ as a function of R performs a random walk (for k-space top-hat filter)
- probability of $\delta > \delta_c$ is given by $\mathrm{erfc}(\delta_c/(\sqrt{2}\sigma(M))$

CDM mass functions



CDM mass function: comoving number density of haloes (redshifts $z = 0, 1, 2, 3$)

- shape of mass function: power law with exponential cut-off
- CDM:
  - hierarchical structure formation: more massive objects form later
  - cut-off scale $M_* \propto D_+(z)^3$ (dark energy influence!)
- normalisation: $\simeq 100$ clusters and $\simeq 10^4$ galaxies in a cube with side length $100\ \mathrm{Mpc}/h$ today ($a = 1$, $z = 0$)

cosmological parameter from cluster surveys

- mass function (comoving number density of haloes of mass M)

$$n(M,z)\mathrm{d}M = \sqrt{\frac{2}{\pi}}\rho_0\Delta(M,z)\frac{\mathrm{d}\ln\sigma(M)}{\mathrm{d}\ln M}\exp\left(-\frac{\Delta^2(M,z)}{2}\right)\frac{\mathrm{d}M}{M^2} \qquad (18)$$

with $\rho_0 = \Omega_m\rho_{\mathrm{crit}}$

- Δ describes the ratio between collapse overdensity and variance of the fluctuation strength on the mass scale M:

$$\Delta(M,z) = \frac{\delta_c(z)}{D_+(z)\sigma(M)} \qquad (19)$$

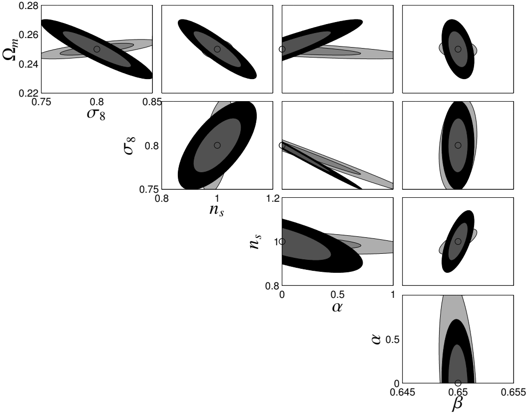
- comoving space is a theoretical construct, we observe redshifts!

$$N(z) = \frac{\Delta\Omega}{4\pi}\frac{\mathrm{d}V}{\mathrm{d}z}\int_{M_{\min}(z)}^{\infty}\mathrm{d}M\; n(M,z) \qquad (20)$$

- comoving volume element, with the angular diameter distance d_A:

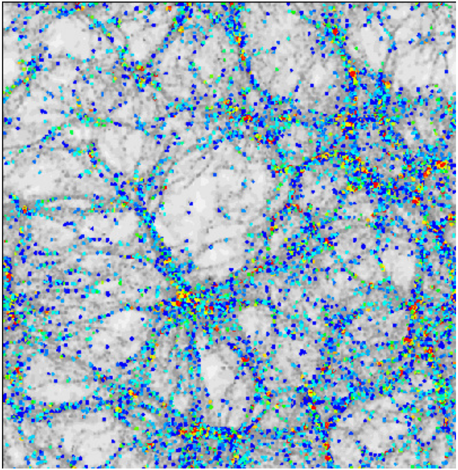
$$\frac{\mathrm{d}V}{\mathrm{d}z} = 4\pi\frac{d_A^2(a)}{a^2H(a)} \qquad (21)$$

cosmological parameter from cluster surveys



cosmological parameters from cluster surveys

galaxy biasing



GIF-simulation, Kaufmann et al.

galaxy bias models

- galaxies trace the distribution of dark matter
- simplest (local, linear, static, morphology and scale-indep.) relation:

$$\frac{\delta n}{\langle n \rangle} = b \frac{\rho}{\langle \rho \rangle} \tag{22}$$

  with **bias parameter** b

- bias models:
  - massive objects are more clustered (larger b) than low-mass objects
  - red galaxies are stronger clustered than blue galaxies
  - bias is slowly time evolving and **decreases**
- physical explanation: galaxies form at local peaks in the dark matter field, and reflect the local matter density directly
- naturally: $\xi_{\mathrm{galaxy}}(r) = b^2 \xi_{\mathrm{CDM}}(r)$ for the above model

question

are there more galaxies if b is larger?

galaxy formation: Jeans instability

- galaxies form by condensation of baryons inside potential wells formed by dark matter
- cooling process: needs to be fast, for overcoming the negative specific heat of a self-gravitating system
- hydrostatic equilibrium: balance **pressure** and **gravity**

$$\frac{\mathrm{d}p}{\mathrm{d}r} = -\frac{GM}{r^2}\rho \tag{23}$$

- collapse: internal pressure smaller than gravity, which happens if M is large, or the temperature small (small pressure)

Jeans mass

Jeans mass is the **minimum mass** for galaxy formation

Jeans-scale: derivation

- initially: spherical gas cloud of radius R and mass M
- compress cloud slightly: pressure wave will propagate through it, and establish new equilibrium
  - pressure equilibration = sound crossing time $t_{\rm sound} = \frac{R}{c_s}$
  - gravitational collapse = free-fall time scale $t_{\rm grav} = \frac{1}{\sqrt{G\rho}}$
- compare time scales
  - $t_{\rm grav} > t_{\rm sound}$ pressure wins, system settles in new equilibrium
  - $t_{\rm grav} < t_{\rm sound}$ gravity wins, system undergoes spherical collapse
- Jeans length $R_J = c_s t_{\rm grav}$ allows to determine Jeans mass M_J:

$$M_J = \frac{4\pi}{3}\rho\left(\frac{R_J}{2}\right)^3 = \frac{\pi}{6}\frac{c_s^3}{G^{1.5}\rho^{0.5}} \tag{24}$$

stability of elliptical galaxies

- stabilisation of elliptical galaxies → **velocity dispersion**
- Jeans equations are 2 coupled nonlinear PDEs for the evolution of collisionless systems
  - first moment: continuity

$$\frac{\partial\rho}{\partial t} + \mathrm{div}(\rho\vec{\upsilon}) = 0 \tag{25}$$

  - second moment: momentum equation

$$\frac{\partial\vec{\upsilon}}{\partial t} + \vec{\upsilon}\nabla\vec{\upsilon} = -\nabla\Phi - \mathrm{div}(\rho\sigma^2) \tag{26}$$

- no viscosity, and velocity dispersion tensor $\sigma_{ij}^2 = \langle\upsilon_i\upsilon_j\rangle - \langle\upsilon_i\rangle\langle\upsilon_j\rangle$ **emulates** (possibly anisotropic) pressure
- gravitational potential: self-consistently derived from Poisson's equation $\Delta\Phi = 4\pi G\rho$, closed system!
- in a virialised elliptical galaxy, σ_{ij} corresponds to $\langle V\rangle$ → **stability**

stability of spiral galaxies

- collisionless fluids can not build up pressure against gravity
- a rotating system can provide force balancing → **centrifugal force**
- spin-up: explained by tidal torquing
- spin-parameter λ

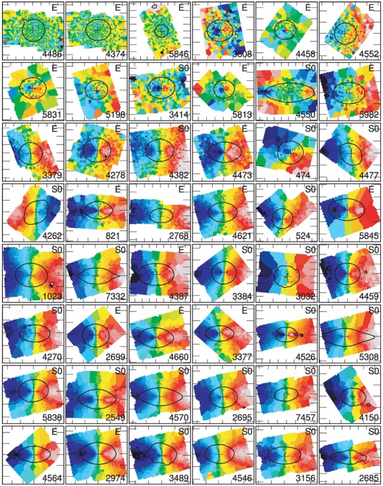
$$\lambda \equiv \frac{\omega}{\omega_0} = \frac{L/(MR^2)}{\sqrt{GM/R^3}} = \frac{L\sqrt{E}}{GM^{5/2}} \tag{27}$$

  - specific angular momentum necessary for rotational support
  - $\lambda \simeq 1/2$ in spirals in ΛCDM cosmologies, rotation is the dominant supporting mechanism

question

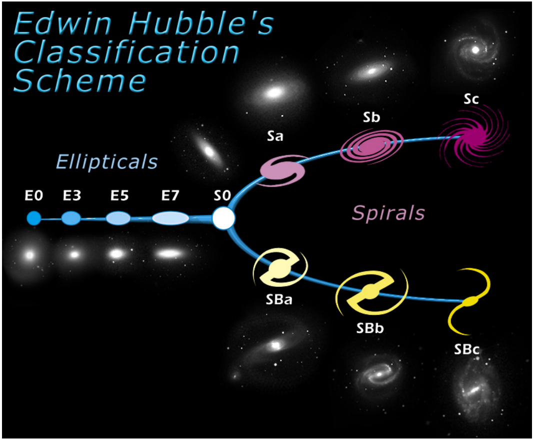
why is the definition of λ sensible?

SAURON observations of galaxies



source: SAURON experiment

galaxy morphologies: ’tuning fork’ diagramme



source: wikipedia

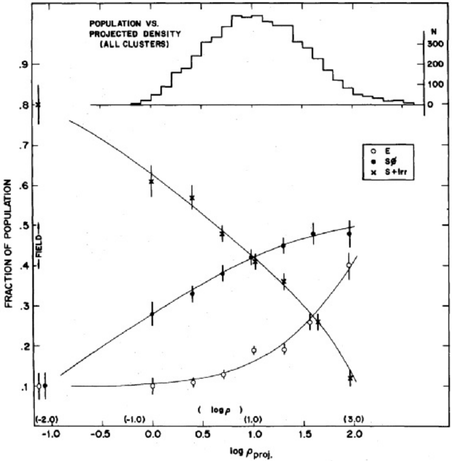
merging of haloes

- contary to Hubble's hypothesis: merging activity and tidal interaction influence galaxy morphologies and convert spirals into ellipticals → **density-morphology relation**
- confusing nomenclature remains:

elliptical	early-type	old stars
spiral	late-type	young stars

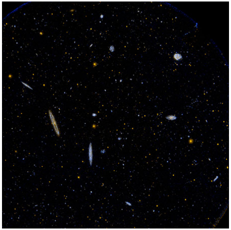
- merging generates heavy haloes from low-mass systems and wipes out the kinematical structure by violent relaxation → **bottom-up structure formation**
- merging activity depends on the cosmology, and causes the mass function to evolve

density-morphology relation



density-morphology relation, source: Dressler et al. (1980s)

galaxy clusters



Perseus cluster (source: NASA/JPL)

Virgo cluster (source: USM)

- largest, gravitationally bound objects, with $M > M_*$
- quasar host structures at high redshift
- historically
  - visual identification (Abell catalogue)
  - need for dark matter: dynamical mass $\gg$ sum of galaxies (Zwicky)
- large clusters have masses of $10^{15} M_\odot/h$ and contain $\sim 10^3$ galaxies

X-ray emission of clusters

- the intra-cluster medium of clusters of galaxies is so hot ($T \simeq 10^7$K) that is produces thermal X-ray radiation
- the plasma is in hydrostatic equilibrium with gravity, therefore the density profile can be computed

$$\frac{\mathrm{d}p}{\mathrm{d}r} = -\frac{GM(r)}{r^2}\rho \rightarrow \frac{k_BT}{m}\frac{\mathrm{d}\rho}{\mathrm{d}r} + \frac{\rho k_B}{m}\frac{\mathrm{d}T}{\mathrm{d}r} = -\frac{GM}{r^2}\rho \tag{28}$$

for ideal gas with $p = \rho k_B T/m$

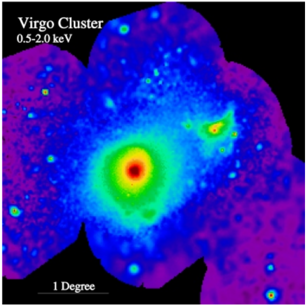
- determination of mass: from measurement of the density and temperature profile:

$$M(r) = -\frac{rk_BT}{Gm}\left(\frac{\mathrm{d}\ln\rho}{\mathrm{d}\ln r} + \frac{\mathrm{d}\ln T}{\mathrm{d}\ln r}\right) \tag{29}$$

question

what can one do if the cluster is not spherically symmetric?

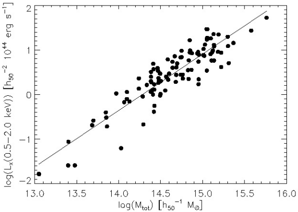
X-ray emission of clusters: ROSAT data



VIRGO cluster as seen by ROSAT

- cluster is in hydrostatic equilibrium
- X-ray emissivity is $\propto \sqrt{T}\rho^2 \rightarrow$ fuzzy blobs

scaling relations



scaling relation between L_X and M from the ROSAT survey

- virial relation allow the prediction of simple **scaling relations**
- valid for fully virialised systems, where the temperature reflects the release in gravitational binding energy
    - potential energy $\langle V\rangle \propto -GM^2/R$
    - size $M \propto R^3 \rightarrow \langle V\rangle \propto -M^{5/3}$
    - kinetic energy $\langle T\rangle \propto TM$
    - virial relation $2\langle T\rangle = -\langle V\rangle \rightarrow T \propto M^{2/3}$
    - X-ray luminosity $L_X \propto M^2\sqrt{T}/R^3 \propto M^{4/3} \propto T^2$

summary

- dark matter objects form by gravitational collapse
- stable solutions are admissible, particles moving inside their own collective potential, typical profiles: NFW, isothermal
- number density and fluctuations statistics can be derived from the power spectrum with the Press-Schechter formalism
- mass function contains cosmological information, in particular Ω_m and σ_8, some sensitivity on w
- presence of baryons: Jeans argument, minimal mass for galaxy formation due to pressure equilibration
- stability of galaxies: rotational stabilisation for spirals, velocity dispersion for ellipticals
- assembly of massive objects by merging
- galaxy clusters: most massive virialised objects, scaling relations