inflation and the cosmic microwave background

cosmology lecture (chapter 9)

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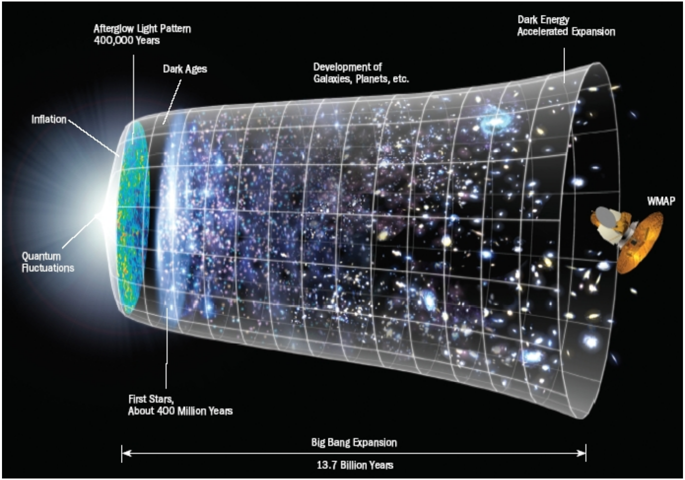
outline

repetition

- Friedmann-Lemaître cosmologies explain the expansion dynamics $H(a)$ of the universe
- fluids influence Hubble-expansion according to their eos parameter
  - $w > -1/3$: expansion slows down
  - $w < -1/3$: expansion accelerates
- dark energy and the cosmological constant can cause accelerated expansion $q < 0$, for sufficiently negative eos $w > -1/3$
- Hubble expansion is an adiabatic process, temperature drops while expanding, $T(a)$ depends on adiabatic index κ
- relic radiation (highly redshifted) from early epochs of the universe
  - big bang nucleosynthesis: neutrino background $T_\nu = 1.95\mathrm{K}$
  - formation of hydrogen atoms: microwave background $T_{\mathrm{CMB}} = 2.75\mathrm{K}$

expansion history of the universe



expansion history of the universe

Planck-scale

- at $a = 0$, $z = \infty$ the metric diverges, and $H(a)$ becomes infinite
- description of general relativity breaks down, quantum effects become important
- relevant scales:
  - quantum mechanics: de Broglie-wave length: $\lambda_\mathrm{QM} = \frac{2\pi\hbar}{mc}$
  - general relativity: Schwarzschild radius: $r_s = \frac{2Gm}{c^2}$
- setting $\lambda_\mathrm{QM} = r_s$ defines the **Planck mass**

$$m_P = \sqrt{\frac{\hbar c}{G}} \simeq 10^{19}\mathrm{GeV}/c^2 \qquad (1)$$

question

how would you define the corresponding Planck length and the Planck time? what are their numerical values?

flatness problem

- construct a universe with matter $w = 0$and curvature $w = -1/3$
- Hubble function

$$\frac{H^2(a)}{H_0^2} = \frac{\Omega_m}{a^3} + \frac{\Omega_K}{a^2} \tag{2}$$

- density parameter associated with curvature

$$\frac{\Omega_K(a)}{\Omega_K} = \frac{H_0^2}{a^{3(1+w)}H^2(a)} = \frac{H_0^2}{a^2H^2(a)} \tag{3}$$

- Ω_K increases always and was smaller in the past

$$\Omega_K(a) = \left(1 + \frac{\Omega_m}{\Omega_K}\frac{1}{a}\right)^{-1} \simeq \frac{\Omega_K}{\Omega_m}a \tag{4}$$

- we know (from CMB observations) that curvature is very small today, typical limits are $\Omega_K < 0.01 \rightarrow$ **even smaller in the past**
  - at recombination $\Omega_K \simeq 10^{-5}$
  - at big bang nucleosynthesis $\Omega_K \simeq 10^{-12}$

horizon problem

- horizon size: light travel distance during the age of the universe

$$\chi_H = c \int \frac{\mathrm{d}a}{a^2 H(a)} \tag{5}$$

- assume $\Omega_m = 1$, integrate from $a_{\mathrm{min}} = a_{\mathrm{rec}} \ldots a_{\mathrm{max}} = 1$

$$\chi_H = 2\frac{c}{H_0}\sqrt{\Omega_m a_{\mathrm{rec}}} = 175\sqrt{\Omega_m}\mathrm{Mpc}/h \tag{6}$$

- comoving size of a volume around a point at recombination inside which all points are in causal contact
- angular diameter distance from us to the recombination shell:

$$d_{\mathrm{rec}} \simeq 2\frac{c}{H_0}a_{\mathrm{rec}} \simeq 5\mathrm{Mpc}/h \tag{7}$$

- angular size of the particle horizon at recombination: $\theta_{\mathrm{rec}} \simeq 2^\circ$
- points in the CMB separated by more than 2° have never been in causal contact $\rightarrow$ **why is the CMB so uniform if there is no possibility of heat exchange?**

inflation: phenomenology

- curvature $\Omega_K \propto$ to the **comoving Hubble radius** $c/(aH(a))$
- if by some mechanism, $c/(aH)$ could decrease, it would drive Ω_K towards 0 and solve the fine-tuning required by the flatness problem
- shrinking comoving Hubble radius:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{c}{aH}\right) = -c\frac{\ddot{a}}{\dot{a}^2} < 0 \rightarrow \ddot{a} > 0 \rightarrow q < 0 \tag{8}$$

- equivalent to the notion of accelerated expansion
- accelerated expansion can be generated by a dominating fluid with sufficiently negative equation of state $w = -1/3$
- horizon problem: fast expansion in inflationary era makes the universe grow from a small, causally connected region

question

what's the relation between deceleration q and equation of state w?

inflaton-driven expansion

- analogous to dark energy, one postulates an **inflaton field** ϕ, with a small kinetic and a large potential energy, for having a sufficiently negative equation of state for accelerated expansion
- pressure and energy density of a homogeneous scalar field

$$p = \frac{\dot{\phi}^2}{2} - V(\phi), \quad \rho = \frac{\dot{\phi}^2}{2} + V(\phi) \tag{9}$$

- Friedmann equation

$$H^2(a) = \frac{8\pi G}{3}\left(\frac{\dot{\phi}^2}{2} + V(\phi)\right) \tag{10}$$

- continuity equation

$$\ddot{\phi} + 3H\dot{\phi} = -\frac{\mathrm{d}V}{\mathrm{d}\phi} \tag{11}$$

slow roll conditions

- inflation can only take place if $\dot{\phi}^2 \ll V(\phi)$
- inflation needs to keep going for a sufficiently long time:

$$\frac{\mathrm{d}}{\mathrm{d}t}\dot{\phi}^2 \ll \frac{\mathrm{d}}{\mathrm{d}t}V(\phi) \rightarrow \ddot{\phi} \ll \frac{\mathrm{d}}{\mathrm{d}\phi}V(\phi) \tag{12}$$

- in this regime, the Friedmann and continuity equations simplify:

$$H^2 = \frac{8\pi G}{3}V(\phi), \quad 3H\dot{\phi} = -\frac{\mathrm{d}}{\mathrm{d}\phi}V(\phi) \tag{13}$$

- conditions are fulfilled if

$$\frac{1}{24\pi G}\left(\frac{V'}{V}\right)^2 \equiv \epsilon \ll 1, \quad \frac{1}{8\pi G}\left(\frac{V''}{V}\right) \equiv \eta \ll 1 \tag{14}$$

- ϵ and η are called **slow-roll parameters**

stopping inflation

- flatness problem: shrinkage by $\simeq 10^{30} \simeq \exp(60) \rightarrow$ **60 e-folds**
- due to the slow-roll conditions, the energy density of the inflaton field is almost constant
- all other fluid densities drop by huge amounts, ρ_m by 10^{90}, ρ_γ by 10^{120}
- eventually, the slow roll conditions are not valid anymore, the effective equation of state becomes less negative, acclerated expansion stops
- **reheating:** couple ϕ to other particle fields, and generate particles from the inflaton's kinetic energy

question

the temperature at the beginning of inflation was equal to the Planck-temperature. what is its value (in Kelvin) and by how much has it dropped until today?

generation of fluctuations

- fluctuations of the inflation field can perturb the distribution of all other fluids
- mean fluctuation amplitude is related to the variance of ϕ
- fluctuations in ϕ perturb the metric, and all other fluids feel a perturbed potential
- relevant quantity

$$\sqrt{\langle\delta\Phi^2\rangle} \simeq \frac{H^2}{V} \tag{15}$$

  which is approximately constant during slow-roll
- Poisson-equation in Fourier-space $k^2\Phi(k) = -\delta(k)$
- variance of density perturbations:

$$|\delta(k)|^2 \propto k^4 \, |\delta\Phi|^2 \propto k^3 P(k) \tag{16}$$

- defines **spectrum** $P(k)$ of the initial fluctuations, $P(k) \propto k^n$ with $n \simeq 1$
- fluctuations are Gaussian, because of the **central limit theorem**

random fields

- random process → probability density $p(\delta)\mathrm{d}\delta$ of event δ
- alternatively: all moments $\langle\delta^n\rangle = \int \mathrm{d}\delta\, \delta^n p(\delta)$
- in cosmology:
  - random events are values of the density field δ
  - outcomes for $\delta(\vec{x})$ form a statistical ensemble at fixed $\vec{x}$
  - ergodic random processes:
    one realisation is consistent with $p(\delta)\mathrm{d}\delta$
- special case: Gaussian random field
  - only **variance** relevant

characteristic function $\phi(t)$

- for a continuous pdf, all moments need to be known for reconstructing the pdf
- reconstruction via **characteristic function** $\phi(t)$ (Fourier transform)

$$\phi(t) = \int \mathrm{d}x p(x) \exp(\mathrm{i}tx) = \int \mathrm{d}x p(x) \sum_n \frac{(\mathrm{i}tx)^n}{n!} = \sum_n \langle x^n \rangle_p \frac{(\mathrm{i}t)^n}{n!} \quad (17)$$

  with moments $\langle x^n \rangle = \int \mathrm{d}x x^n p(x)$
- Gaussian pdf is special:
  - all moments exist! (counter example: Cauchy pdf)
  - all odd moments vanish
  - all even moments are expressible as products of the variance
  - σ is enough to statistically reconstruct the pdf
  - pdf can be differentiated arbitrarily often (Hermite polynomials)
- funky notation: $\phi(t) = \langle \exp(\mathrm{i}tx) \rangle$

cosmic microwave background

- inflation has generated perturbations in the distribution of matter
- the hot baryon plasma feels fluctuations in the distribution of (dark) matter by gravity
- at the point of (re)combination:
  - hydrogen atoms are formed
  - photons can propagate freely
- perturbations can be observed by two effects:
  - plasma was not at rest, but flowing towards a potential well → Doppler-shift in photon temperature, depending to direction of motion
  - plasma was residing in a potential well → gravitational redshift
- between the end of inflation and the release of the CMB, the density field was growth **homogeneously** → all statistical properties of the density field are conserved
- testing of inflationary scenarios is possible in CMB observations

formation of hydrogen: (re)combination

- temperature of the fluids drops during Hubble expansion
- eventually, the temperature is sufficiently low to allow the formation of hydrogen atoms
- but: high photon density (remember $\eta_B = 10^9$) can easily reionise hydrogen
- Hubble-expansion does not cool photons fast enough between recombination and reionisation
- neat trick: recombination takes place by a 2-photon process

question

at what temperature would the hydrogen atoms form if they could recombine directly? what redshift would that be?

question

why do we observe a continuum spectrum from the formation of atoms?

CMB motion dipole

- the most important structure on the microwave sky is a dipole
- CMB dipole is interpreted as a relative motion of the earth
- CMB dispole has an amplitude of $10^{-3}K$, and the peculiar velocity is $\beta = 371\mathrm{km/s} \cdot c$

$$T(\theta) = T_0\,(1 + \beta\cos\theta) \tag{18}$$

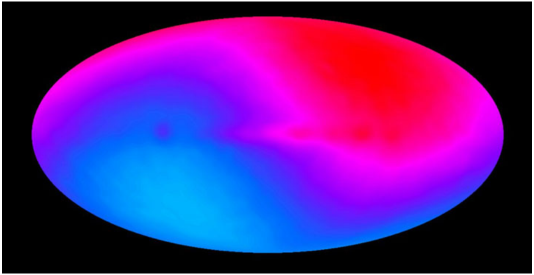
question

is the Planck-spectrum of the CMB photons conserved in a Lorentz-boost?

question

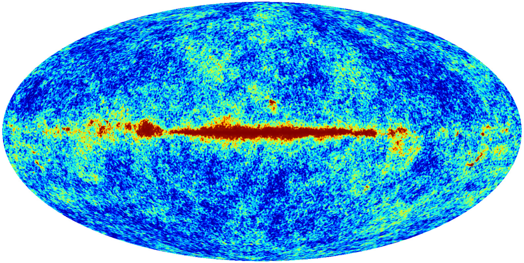
would it be possible to distinguish between a motion dipole and an intrinsic CMB dipole?

CMB dipole



source: COBE

subtraction of motion dipole: primary anisotropies



source: PLANCK simulation

CMB angular spectrum

- analysis of fluctuations on a sphere: decomposition in $Y_{\ell m}$

$$T(\theta) = \sum_\ell \sum_m t_{\ell m} Y_{\ell m}(\theta) \quad \leftrightarrow \quad t_{\ell m} = \int \mathrm{d}\Omega\, T(\theta) Y^*_{\ell m}(\theta) \tag{19}$$

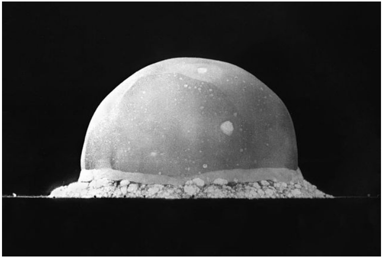
- spherical harmonics are an orthonormal basis system
- average **fluctuation variance** on a scale $\ell \simeq \pi/\theta$

$$C(\ell) = \langle |t_{\ell m}|^2 \rangle \tag{20}$$

- averaging $\langle \ldots \rangle$ is a hypothetical ensemble average. in reality, one computes an estimate of the variance,

$$C(\ell) \simeq \frac{1}{2\ell + 1} \sum_{m=-\ell}^{m=+\ell} |t_{\ell m}|^2 \tag{21}$$

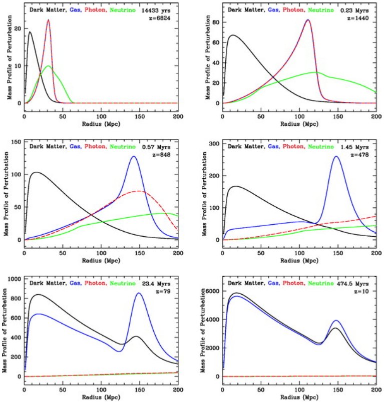
standard ruler principle



trinity nuclear test, 16 milli-seconds after explosion

- physical size: combine
  1. time since explosion
  2. velocity of fireball
- distance: combine
  1. physical size
  2. angular size

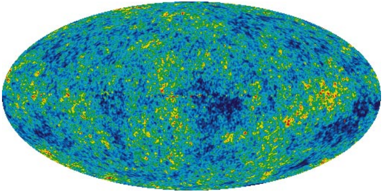
formation of baryon acoustic oscillations



evolution of a single perturbation (source: Eisenstein, Seo and Hu (2005))

- from a pointlike perturbation, a spherical wave travels in the photon-baryon-plasma
- propagation stops when atoms form

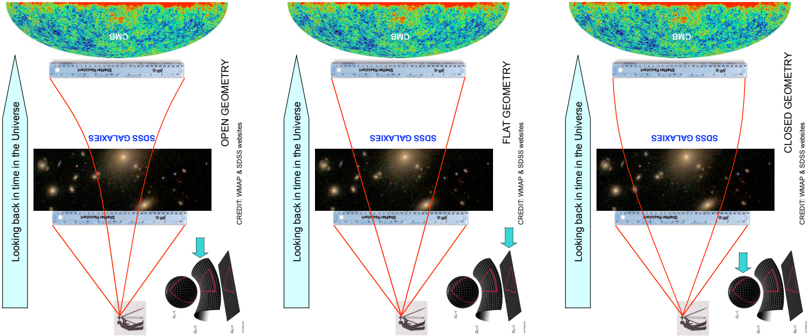
cosmic microwave background: standard ruler



all-sky map of the cosmic microwave background, WMAP

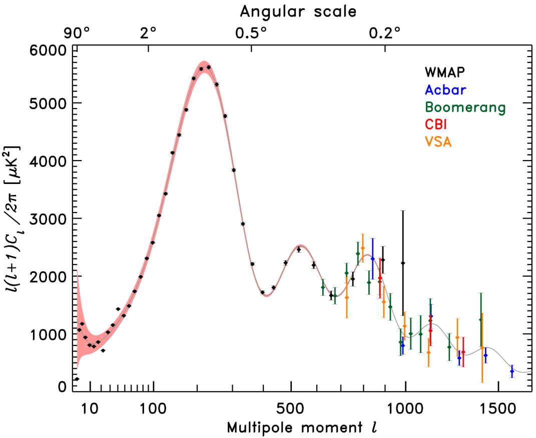
- hot and cold patches of the CMB have a typical physical size, related to the horizon size at the time of formation of hydrogen atoms
- idea: physical size and apparent angle are related, redshift of decoupling known

standard ruler: measurement principle (Eisenstein)



- curvature can be well constrained
- assumption: galaxy bias understood, nonlinear structure formation not too important

parameter sensitivity of the CMB spectrum



source: WMAP

features in the CMB spectrum

- predicting the spectrum $C(\ell)$ is **very complicated**
- perturbations in the CMB photons $n \propto T^3$, $u \propto T^4$, $p = u/3 \propto T^4$:

$$\frac{\delta n}{n_0} = 3\frac{\delta T}{T} \equiv \Theta, \quad \frac{\delta u}{u_0} = 4\Theta = \frac{\delta p}{p_0} \tag{22}$$

- continuity and Euler equations:

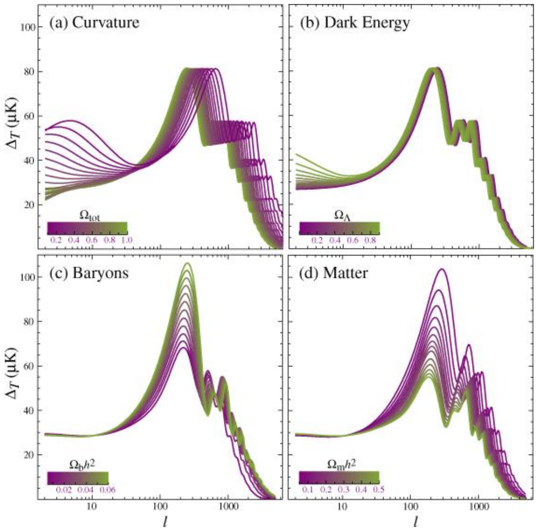
$$\dot{n} = n_0 \mathrm{div} \upsilon = 0, \quad \dot{\upsilon} = -c^2 \frac{\nabla p}{u_0 + p_0} + \nabla\delta\Phi \tag{23}$$

- use $u_0 + p_0 = 4/3u_0 = 4p_0$
- combine both equations

$$\ddot{\Theta} - \frac{c^2}{3}\Delta\Theta + \frac{1}{3}\Delta\delta\Phi = 0 \tag{24}$$

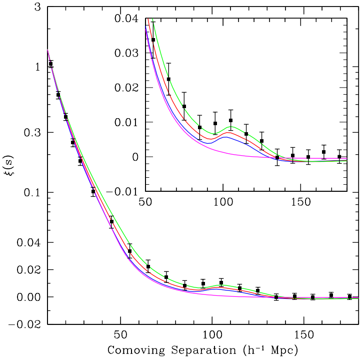
- identify two mechanisms:
  - oscillations may occur, and photons experience **Doppler shifts**
  - photons feel fluctations in the potential: **Sachs-Wolfe effect**

parameter sensitivity of the CMB spectrum



source: Wayne Hu

baryon acoustic oscillations in the galaxies



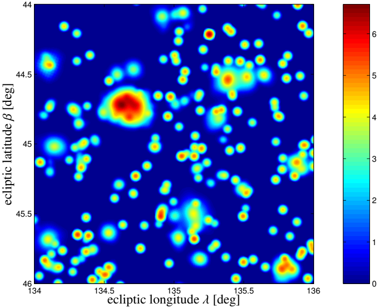
pair density $\xi(r)$ of galaxies as a function of separation r

- baryon acoustic oscillations: the (pair) density of galaxies is enhanced at a separation of about $100\mathrm{Mpc}/h$ comoving
- idea: angle under which this scale is viewed depends on redshift

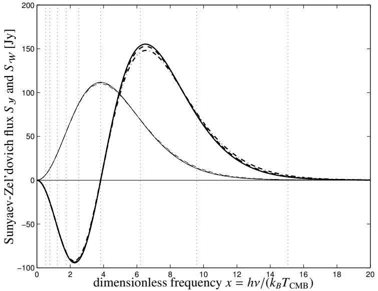
secondary CMB anisotropies

- CMB photons can do interactions in the cosmic large-scale structure on their way to us
- two types of interaction: **Compton-collisions** and **gravitational**
- consequence: secondary anisotropies
- study of secondaries is very interesting: observation of the growth of structures possible, and precision determination of cosmological parameters
- all effects are in general important on **small angular scales** below a degree

thermal Sunyaev-Zel'dovich effect



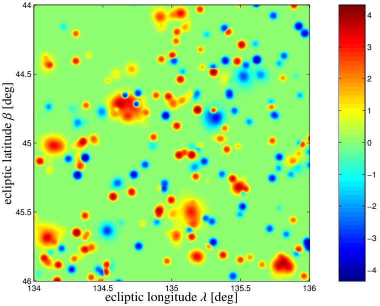
thermal SZ sky map



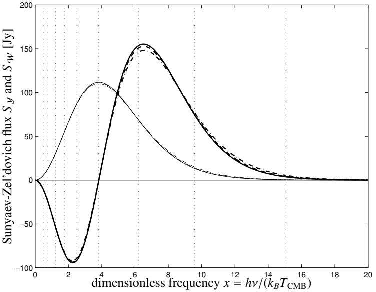
CMB spectrum distortion

- Compton-interaction of CMB photons with **thermal** electrons in clusters of galaxies
- characteristic redistribution of photons in energy spectrum

kinetic Sunyaev-Zel'dovich/Ostriker-Vishniac effect



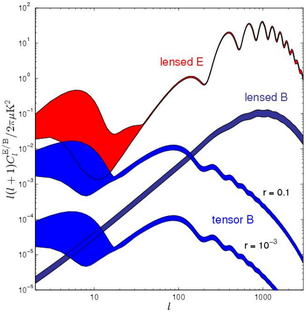
thermal SZ sky map



CMB spectrum distortion

- Compton-interaction of CMB photons with electrons in **bulk flows**
- increase/decrease in CMB temperature according to direction of motion

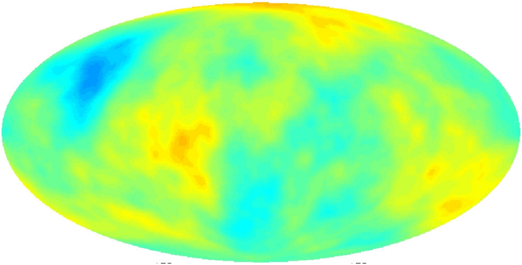
CMB lensing



source: A. Lewis, A. Challinor

- gravitational deflection of CMB photons on potentials in the cosmic large-scale structure
- CMB spots get distorted, and their fluctuation statistics is changed, in particular the polarisation

integrated Sachs-Wolfe effect



source: B. Barreiro

- gravitational interaction of photons with time-evolving potentials
- higher-order effect on photon geodesics in general relativity

summary

- inflation:
  - solves flatness-problem
  - solves horizon-problem
  - generates perturbations in the density field
- perturbations are Gaussian, and can be described by a correlation function $\xi(r)$ or a power spectrum $P(k)$
- perturbations have a scale-free spectrum $P(k) \propto k^{n_s}$ with $n_s \simeq 1$
- Meszaros-effect changes spectrum to $P(k) \propto k^{n_s-4}$ on small scales
- normalisation of the CDM spectrum $P(k)$ by the parameter σ_8
- cosmic microwave background probes inflationary perturbations
  - dynamics of the plasma in potential fluctuations
  - precision determination of cosmological parameters
  - secondary effects influence the CMB on small scales (gravitational lensing, Sunyaev-Zel'dovich effect, integrated Sachs-Wolfe effect)