thermal history of the universe and big bang nucleosynthesis

cosmology lecture (chapter 8)

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outline

repetition

- Friedmann-Lemaître cosmological models:
  - homogeneous and isotropic metric (Robertson-Walker line element)
  - homogeneous and ideal fluids, described by ρ and w
- evolving metric, described by Hubble function $H(a)$
  $\rightarrow$ distance measures, interesting features
- geometrical measurements (SN distances) constrain densities
- cosmological fluids, influence $H(a)$ by their eos parameter w
- preferred model: ΛCDM, **flat and accelerating**
  - radiation $\Omega_\gamma \simeq 10^{-4}$
  - matter $\Omega_m = 0.25$
  - curvature $\Omega_k \simeq 0$
  - cosmological constant $\Omega_\Lambda = 0.75$

thermal history of the universe

- temperature of fluids drop while universe expands
- 3 important stages
  - temperature is high enough to allow nuclear reactions
    → big bang nucleosynthesis ($z \simeq 10^{10}$)
  - densities of radiation and matter are equal
    → matter-radiation equality ($z \simeq 10^{4}$)
  - temperature is high enough to ionise hydrogen
    → cosmic microwave background ($z \simeq 10^{3}$)

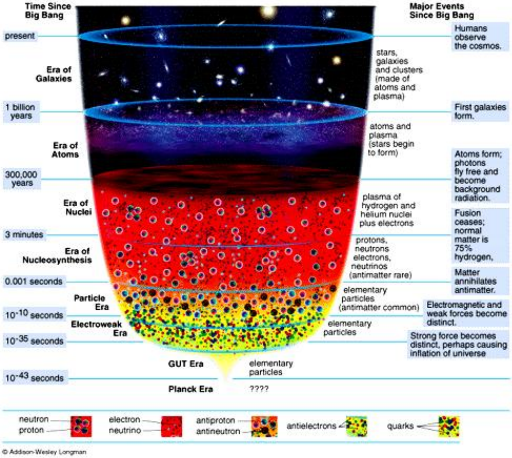
question

would the universe heat up again if contracting?

question

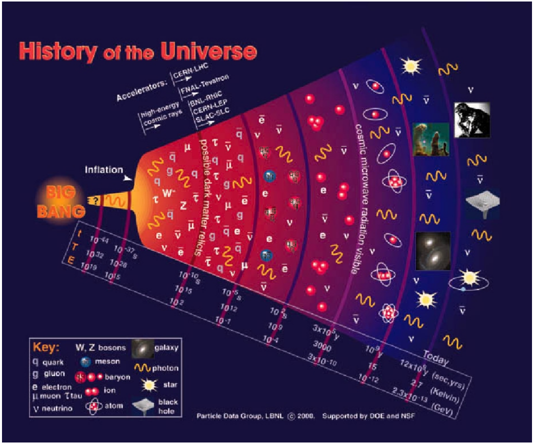
can you derive the redshift (or scale factor) for $\Omega_m(z) = \Omega_\gamma(z)$?

thermal history of the universe: overview



source: Addison-Wesley

thermal history of the universe: particle interactions



source: particle data group

adiabaticity

- 3 basic assumptions
  - **adiabatic** expansion, no heat flux into the universe
  - thermal **equilibration** possible despite Hubble expansion
  - fluids are **ideal**, no long-range interactions between particles
- entropy
  - $\mathrm{d}Q = 0$ does not say anything about entropy generation
  - both reversible and irreversible processes are possible
  - dominating contribution to the entropy are the CMB photons
  - irreversible processes happen (formation of objects and relaxation towards final state!), but their entropy contribution is negligible
  - in this sense $\mathrm{d}Q = 0$ implies $\mathrm{d}S = 0$, and entropy is conserved in adiabatic expansion

thermodynamical distributions

- spin-statistics theorem: in equilibrium,
  - integer spin particles are Bose-Einstein distributed
  - half-integer spin particles are Fermi-Dirac distributed
- in cosmology, we consider
  - photons (spin 1)
  - neutrinos (spin 1/2)
  - dark matter particles and baryons (scalar, spin 0)
- Bose-Einstein distribution

$$n(p,T) = \frac{g}{(2\pi\hbar)^3}\int_0^\infty \mathrm{d}p\,\frac{4\pi p^2}{\exp(\epsilon(p)/k_BT) - 1} \tag{1}$$

- Fermi-Dirac distribution

$$n(p,T) = \frac{g}{(2\pi\hbar)^3}\int_0^\infty \mathrm{d}p\,\frac{4\pi p^2}{\exp(\epsilon(p)/k_BT) + 1} \tag{2}$$

- with the energy-momentum relation $\epsilon(p)$

properties of quantum gases

- properties of ensembles can be derived by computing the partition sum using the fundamental distribution
- thermodynamic potentials follow by logarithmic differentiation
- for **bosons** (integer spin)

$$n_B = \frac{g_B \zeta(3)}{\pi^2}\left(\frac{k_B T}{\hbar c}\right)^3, u_B = \frac{g_B \pi^2}{30}\frac{(k_B T)^4}{(\hbar c)^3}, s_B = \frac{g_B 2\pi^2}{45}\left(\frac{k_B T}{\hbar c}\right)^3 \tag{3}$$

- for **fermions** (half-integer spin)

$$n_F = \frac{3}{4}\frac{g_F}{g_B} n_B, u_F = \frac{7}{8}\frac{g_F}{g_B} u_B, s_F = \frac{7}{8}\frac{g_F}{g_B} s_B \tag{4}$$

question

what is the density of CMB photons, temperature $T_{\mathrm{CMB}} = 2.726\mathrm{K}$?

temperature and Hubble expansion

- Hubble expansion is an **adiabatic** process $\mathrm{d}Q = 0$
- adiabatic equation: $V^{\kappa-1}T =$ const with adiabatic index $\kappa \equiv c_p/c_V$
- early times: universe is filled with photons $\kappa = 4/3$ (relativistic gas)

$$T \propto V^{-1/3} \propto a^{-1} \tag{5}$$

- late times: universe is filled with (dark) matter $\kappa = 5/3$ (classical gas)

$$T \propto V^{-2/3} \propto a^{-2} \tag{6}$$

question

how would the temperature vary if the universe was filled with molecular gas?

question

if the universe was filled with an ideal gas, how would pressure depend on scale factor?

temperature and Hubble expansion

question

in the real Universe, there are many more photons than baryons, such that the baryon temperature follows the photon temperature until a redshift of $z \simeq 100$. What is the temperature of the baryons today, and what would this number be if you would just let them cool with the adiabatic law?

(chemical) reactions and Hubble expansion

- thermal equilibrium: maintained by particle collisions
- comparison of 2 time-scales: collision t_{coll} and expansion t_{Hubble}
  - $t_{\mathrm{coll}} < t_{\mathrm{Hubble}}$: relaxation, thermal equilibrium can be maintained
  - $t_{\mathrm{Hubble}} < t_{\mathrm{coll}}$: system drops out of equilibrium
- Hubble expansion time-scale is given by (radiation domination)

$$t_{\mathrm{Hubble}} = H^{-1}(a) = a/\dot{a} \simeq (G\rho)^{-1/2} \propto a^2 \tag{7}$$

- collision rate:

$$\Gamma = n\langle\sigma\upsilon\rangle \rightarrow t_{\mathrm{coll}} = \Gamma^{-1} \propto a^3 \tag{8}$$

- early times: $t_{\mathrm{Hubble}}/t_{\mathrm{coll}} \propto a^{-1}$, thermal equilibrium easily maintained
- late times: equilibrium can break down

freeze-out

- continuity equation : $\dot{n} + \mathrm{div}(n\vec{\upsilon}) = 0$
- with isotropic Hubble flow $\vec{\upsilon} = H\vec{r}$ and $\mathrm{div}\vec{r} = 3$: $\dot{n} + 3Hn = 0$
- modification: presence of collisions and particle creation:

$$\dot{n} + 3Hn = -\Gamma n + S = -\Gamma n\left(1 - \frac{n_T^2}{n^2}\right) \tag{9}$$

  - collision rate $\Gamma = n\langle\sigma\upsilon\rangle$
  - particle creation from thermal particles $S = \langle\sigma\upsilon\rangle n_T^2$

- introduce comoving number density $N = na^3$

$$\dot{N} = -\Gamma N\left(1 - \frac{N_T^2}{N^2}\right) \tag{10}$$

with $\dot{N} = a^3(3Hn + \dot{n})$

freeze-out

- comoving continuity equation for particle density

$$\frac{\mathrm{d}\ln N}{\mathrm{d}\ln a} = -\frac{\Gamma}{H}\left(1 - \frac{N_T^2}{N^2}\right) \tag{11}$$

- comparison of the two time-scales $t_{\mathrm{coll}} = \Gamma^{-1}$ and $t_{\mathrm{Hubbe}} = H^{-1}$
- if thermalisation is complete, $N_T = N$, stays thermalised
- if $N_T \neq N$, thermalisation takes place if $\Gamma \ll H$
- **but if $\Gamma \ll H$, the particles can not equilibrate $\rightarrow$ freeze-out**
- for relativistic particles, $n \propto a^{-3}$ and $N = a^3 n = \mathrm{const}$
  $\rightarrow$ thermal distribution maintained even after freeze-out

question

what is the equilibrium ratio between protons and neutrons in the reaction $p + e^- \leftrightarrow n + \nu_e$ at 0.1 MeV? at 1 MeV? at 10 MeV?

cosmic neutrino background

- (electro)weak reactions $\nu + \bar{\nu} \leftrightarrow e^+ + e^- \leftrightarrow 2\gamma$ thermalise νs
  - freeze out temperature of neutrinos $T_\nu \simeq 10^{10.5}$K
  - freeze out temperature of photons $T_\gamma \simeq 10^{10}$K
- e^+/e^--annihilation dumps energy into photons, but not neutrinos
- entropy is conserved in freeze-out: $s'_{e^+} + s'_{e^-} + s'_\gamma = s_\gamma$
- entropy of species differs only by fermionic factor $7/8$: $s'_{e^\pm} = 7/8 s'_\gamma$
- entropy $\propto T^3$, therefore we get for the temperature after annihilation:

$$\left(2\frac{7}{8} + 1\right)(T')^3 = T^3 \rightarrow T \simeq 1.4T' \qquad (12)$$

- γ-temperature is 40% higher compared to ν-temperature

question

what's the temperature of the ν-background at $z = 0$? at $z = 1$?

baryon-photon ratio

- measurements (CMB spectrum X-ray observation of clusters) suggest that the baryon density is low, $\Omega_b = 0.04$
- if all baryons are hydrogen atoms, the number density of baryons is

$$n_b = \frac{\rho_b}{m_p} = \frac{\Omega_b}{m_p}\frac{3H_0^2}{8\pi G} \simeq 1.1\times 10^{-5}\Omega_b h^2 \mathrm{cm}^{-3} \tag{13}$$

- define baryon to photon ratio η

$$\eta \equiv \frac{n_b}{n_\gamma} = 2.7\times 10^{-8}\Omega_b h^2 \simeq 10^{-9} \tag{14}$$

- there are more than 10^9 photons per baryon → dominate entropy

question

show that n_b and n_γ scale identically with temperature T! use a physical argument or properties of the Planck spectrum.

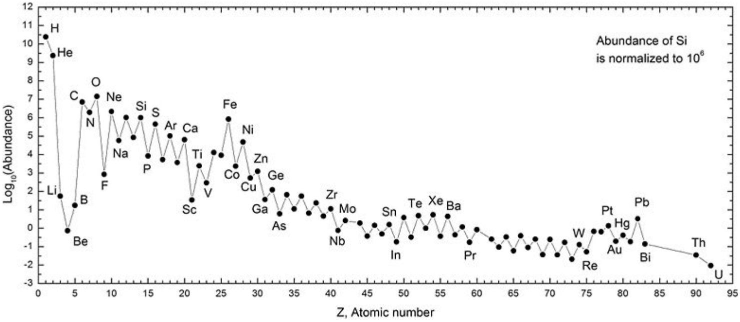
synthesis of elements: overview

- at redshift $z = 10^{10}$, the equilibrium temperature is $k_B T = 1$ MeV, and the universe is filled with a plasma of photons, neutrinos, protons and neutrons
- thermonuclear reactions can produce light elements
  $\rightarrow$ **big bang nucleosynthesis**
- 4 relevant processes for element production
  - stellar nucleosynthesis (thermonuclear burning in stars)
  - spallation (collisions of nuclei in the ISM)
  - explosive nucleosynthesis (heavy element production in supernovae)
  - big bang nucleosynthesis (element production in the early universe)

question

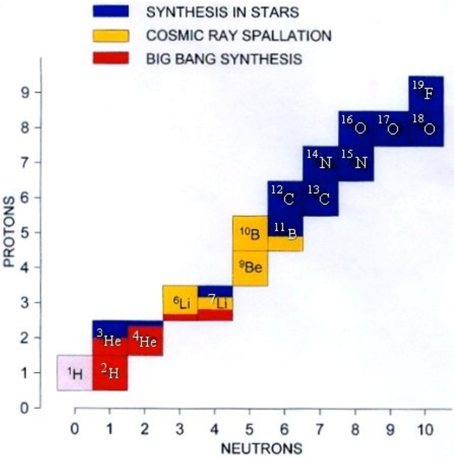
what sets the ranges in which each process is effective?

synthesis of elements: overview



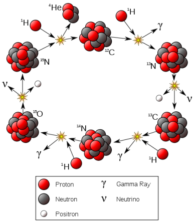
source: wikipedia

synthesis of elements: isotopes

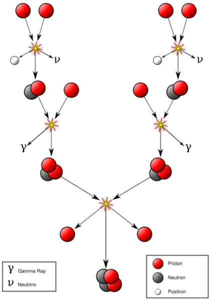


source: universe review

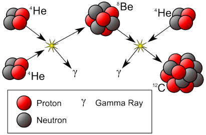
synthesis of elements 1: stellar nucleosynthesis



CNO-cycle



pp-chain



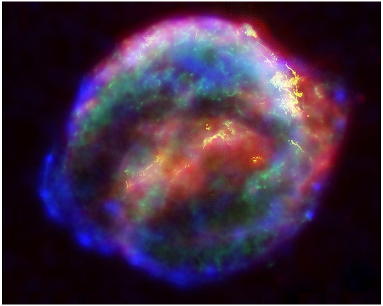
triple α fusion

- stellar nucleosynthesis: thermal reactions at 10^7K
- fusion up to iron/nickel, heavier elements can not be produced exothermally

synthesis of elements 2: spallation

- big bang nucleosynthesis produces
  - up to lithium ($Z = 3$), no stable isotopes
  - stars from carbon on ($Z = 6$), triple α fusion
- what about the bulk of lithium, beryllium ($Z = 4$) and boron ($Z = 5$)
- production by **spallation** in the interstellar medium
- collision between nuclei induces fission
- fragments are light nuclei with low Z

synthesis of elements 3: explosive nucleosynthesis



source: wikipedia, Chandra X-ray image

- element production in supernovae
- heavier elements than iron/nickel can be generated

synthesis of elements 4: big bang nucleosynthesis

- at a temperature of $k_B T \simeq 1$GeV, protons and neutrons transform by β-processes

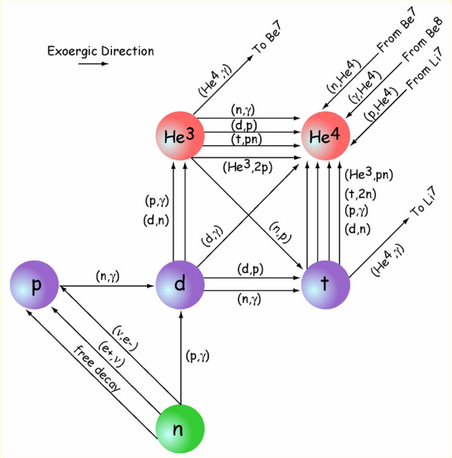
$$p + e^{-} \leftrightarrow n + \nu_e \tag{15}$$

- thermal equilibrium is maintained until β-interaction freezes out at $k_B T \simeq 1$MeV
- nucleosynthesis can take place, by the reactions

$$n + p \rightarrow {}^{2}H + \gamma \tag{16}$$
$${}^{2}H + {}^{2}H \rightarrow {}^{3}He + n \tag{17}$$
$${}^{3}He + {}^{2}H \rightarrow {}^{4}He + p \tag{18}$$
$${}^{4}He + {}^{3}H \rightarrow {}^{7}Li + \gamma \tag{19}$$
$$\tag{20}$$

- absence of stable isotopes with weights $A = 5$ and $A = 8$ make production of heavier elements impossible

nucleosynthesis networks



source: Maldonaldo + Timmes

big bang nucleosynthesis

- deuterium production $n + p \rightarrow {}^2H + \gamma$ would take place at $k_B T \simeq 2$MeV
- high photon density destroys deuterium by **photodissociation** until $k_B T \simeq 100$keV
- deuterum production sets in effectively 3 minutes after the big bang/inflation
- **Gamov**-criterion: deuterium abundance is crucial
  - too much ${}^2H$: neutrons are locked up, no heavier elements can form
  - too little ${}^2H$: synthesis chain can not continue
- ideal deuterium production rate

$$n_b \langle \sigma \upsilon \rangle \, \Delta t \simeq 1 \tag{21}$$

with the time Δt for nucleosynthesis

big bang nucleosynthesis: abundance predictions

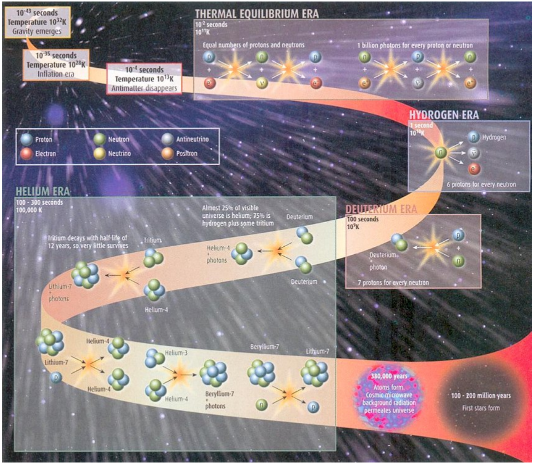
- neutron abundance:
  - neutrons and protons are in equilibrium until $k_B T = 1$MeV
  - fusion for the production of deuterium sets in at $k_B T = 100$keV
- in the meantime, neutrons decay with the half life of $\simeq 15$ minutes
- resulting neutron-proton ratio

$$\frac{n_n}{n_p} \simeq \frac{1}{7} \tag{22}$$

- helium abundance
  - deuterium absorbs neutrons into helium (high binding energy)
- resulting helium mass fraction

$$Y_p \simeq \frac{2n_n}{n_p + n_n} = \frac{1}{4} \tag{23}$$

nucleosynthesis: summary

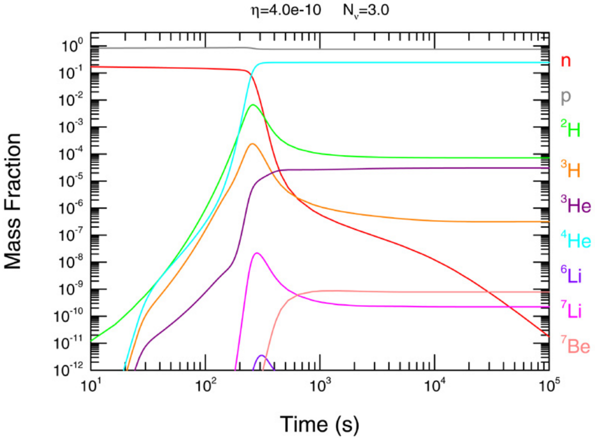


source: universe review

dependence of abundance on η

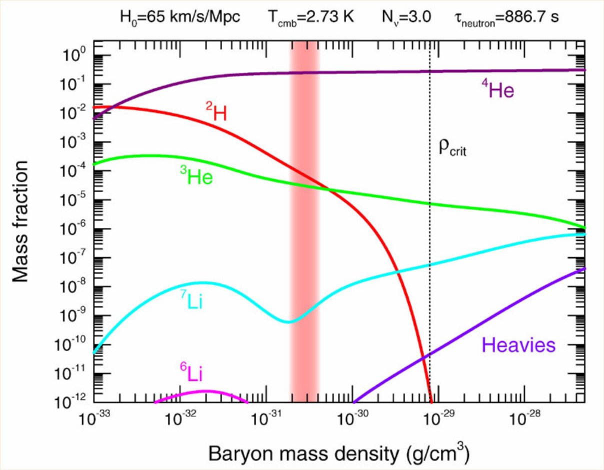
- if η increases (less photons):
  - nucleosynthesis starts earlier, more neutrons decay, more deuterium is produced $\rightarrow Y_p$ increases
  - ^2H adn ^3He are consumed by fusion $\rightarrow$ abundances decrease
  - at low η, ^7Li is destroyed by protons $\rightarrow$ lithium-valley

nucleosynthesis: results 1



source: Maldonaldo + Timmes

nucleosynthesis: results 2



source: Maldonaldo + Timmes

observations

observations are difficult because light elements are produced and consumed in stars

- all isotopes are observed spectroscopically:
  - ${}^2H$: hyperfine transitions
  - ${}^3He$: hyperfine transitions
  - ${}^4He$: optical tranisions while recombinating in HII-regions
  - ${}^7Li$: optical transitions, in cool, low-mass stars
- best probe: ${}^2H$, because it depends on η in a simple way and is only destroyed in stellar fusion

$$\frac{n_{{}^2H}}{n_H} \simeq 3.5 \times 10^5 \tag{24}$$

summary

- ideal fluids undergo an adiabatic process in Hubble expansion and lower their temperature
- collision time-scale determines if a fluid can be thermal
- at high temperatures, the thermonuclear production of light elements from hydrogen is possible
- helium and lithium can be produced in big bang nucleosynthesis, but not heavier elements
- observation of primordial element abundances is possible, but very difficult
- relic radiation from the BBN era is the cosmic neutrino background, with a equilibrium temperature of 1.95K
- photons are the dominating contribution to the entropy, entropy generation is almost negligible